



1- Write the duals of the following problems:

(a) Maximize $Z = -5 X_1 + 2 X_2$

Subject To:

$$\begin{aligned} -X_1 + X_2 &\leq -3 \\ 2X_1 + 3X_2 &\leq 5 \\ X_1, X_2 &\geq 0 \end{aligned}$$

(b) Minimize $Z = 6 X_1 + 3 X_2$

Subject To:

$$\begin{aligned} 6X_1 - 3X_2 + X_3 &\geq 2 \\ 3X_1 + 4X_2 + X_3 &\geq 5 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

(c) Maximize $Z = 5 X_1 + 6 X_2$

Subject To:

$$\begin{aligned} X_1 + 2X_2 &= 5 \\ -X_1 + 5X_2 &\geq 3 \\ X_1 &\text{ unrestricted} \\ X_2 &\geq 0 \end{aligned}$$

(d) Minimize $Z = 3 X_1 + 4 X_2 + 6 X_3$

Subject To:

$$\begin{aligned} X_1 + X_2 &\geq 10 \\ X_1, X_3 &\geq 0 \\ X_2 &\geq 0 \end{aligned}$$

(e) Maximize $Z = X_1 + X_2$

Subject To:

$$\begin{aligned} 2X_1 + X_2 &= 5 \\ 3X_1 - X_2 &= 6 \\ X_1, X_2 &\text{ unrestricted} \end{aligned}$$

2 – Consider the following linear program:

Maximize $Z = 5 X_1 + 2 X_2 + 3 X_3$

Subject To:

$$\begin{aligned} X_1 + 5X_2 + 2X_3 &= 30 \\ X_1 - 5X_2 - 6X_3 &\leq 40 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

The Optimal solution is given by:

Basic	X_1	X_2	X_3	R	X_4	Solution
Z	0	23	7	5+M	0	150
X_1	1	5	2	1	0	30
X_4	0	-10	-8	-1	1	10

Write the dual problem and find its optimal solution from the optimal primal tableau.

3 – Consider the following linear program :

$$\text{Maximize } Z = X_1 + 5 X_2 + 3 X_3$$

Subject To :

$$X_1 + 2 X_2 + X_3 = 3$$

$$2 X_1 - X_2 = 4$$

$$X_1, X_2, X_3 \geq 0$$

Using a starting solution consisting of X_3 and an artificial variable R in the second constraint, we obtain the following optimum tableau.

Basic	X_1	X_2	X_3	R	Solution
Z	0	-2	0	-1+M	5
X_3	1	5/2	1	-1/2	1
X_4	1	-1/2	0	1/2	2

Write the dual problem and find its optimal solution from the optimal primal tableau.

4 – Consider the following linear program:

$$\text{Maximize } Z = 2 X_1 + 4 X_2 + 4 X_3 - 3 X_4$$

Subject To:

$$X_1 + X_2 + X_3 = 4$$

$$X_1 + 4 X_2 + X_4 = 8$$

$$X_1, X_2, X_3, X_4 \geq 0$$

By using X_3 and X_4 as starting variables, the optimum tableau is given by :

Basic	X_1	X_2	X_3	X_4	Solution
Z	2	0	0	3	16
X_3	3/4	0	1	-1/4	2
X_2	1/4	1	0	1/4	2

- (a) –Write the dual problem and find its solution from the optimal primal tableau.
(b) –Use the dual problem to verify that the basic solution (X_1, X_2) is not optimal.

5 – Solve the following problem by considering its dual :

$$\text{Minimize } Z = 5 X_1 + 6 X_2 + 3 X_3$$

Subject To :

$$5 X_1 + 5 X_2 + 3 X_3 \geq 50$$

$$X_1 + X_2 - X_3 \geq 20$$

$$7 X_1 + 6 X_2 - 9 X_3 \geq 30$$

$$5 X_1 + 5 X_2 + 5 X_3 \geq 35$$

$$2 X_1 + 4 X_2 - 15 X_3 \geq 10$$

$$12 X_1 + 10 X_2 \geq 90$$

$$X_2 - 10 X_3 \geq 20$$

$$X_1, X_2, X_3 \geq 0$$

Compare the number of constraints in the two problems.

6 – Find the optimal objective value of the following problem by inspecting only its dual. (Do not solve the dual by the simplex method).

$$\text{Minimize } Z = 10 X_1 + 4 X_2 + 5 X_3$$

Subject To :

$$5 X_1 - 7 X_2 + 3 X_3 \geq 50$$

$$X_1, X_2, X_3 \geq 0$$

7 – Consider the primal problem :

$$\text{Maximize } Z = 3 X_1 + 2 X_2 + 5 X_3$$

Subject To :

$$X_1 + 2 X_2 + X_3 \leq 500$$

$$3 X_1 + 2 X_3 \leq 460$$

$$X_1 + 4 X_2 \leq 420$$

$$X_1, X_2, X_3 \geq 0$$

Write the dual problem for the primal . Without carrying out the simplex method computations on either or the dual problem , estimate a range for the optimum value of the objective function.

8 – The following explicit rules are often listed in most operation research and linear programming books for constructing the dual problem. Show that these rules are subsumed by the general definition given :

Maximization problem		Minimization problem
<u>Constraints</u>		<u>Variables</u>
$\geq$	∞	≤ 0
$\leq$	∞	≥ 0
$=$	∞	Unrestricted
<u>Variables</u>		<u>Constraints</u>
≥ 0	∞	$\geq$
≤ 0	∞	$\leq$
Unrestricted	∞	$=$

9 – True or False ?

- The dual problem yields the original primal .
- If the primal constraint is originally in equation form, the corresponding dual variable is necessarily unrestricted.
- If the primal constraint is of the type $\leq$, the corresponding dual variable will be non negative (non positive) depending on whether the primal objective is maximization (minimization).
- If the primal constraint is of the type $\geq$, the corresponding dual variable will be non negative (non positive) depending on whether the primal objective is minimization (maximization).
- An unrestricted primal variable will result in an equality dual constraint.

10 – Estimate a range for the optimal objective value for each of the following LPs:

(a) Minimize $Z = 5 X_1 + 2 X_2$

Subject To :

$$\begin{aligned} X_1 - X_2 &\geq 3 \\ 2 X_1 + 3 X_2 &\geq 5 \\ X_1, X_2 &\geq 0 \end{aligned}$$

(b) Maximize $Z = X_1 + 5 X_2 + 3 X_3$

Subject To :

$$\begin{aligned} X_1 + 2 X_2 + X_3 &= 3 \\ 2 X_1 - X_2 &= 4 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

(c) Maximize $Z = 2 X_1 + X_2$

Subject To :

$$\begin{aligned} X_1 + X_2 &\leq 10 \\ 2 X_1 &\leq 40 \\ X_1, X_2 &\geq 0 \end{aligned}$$

(d) Maximize $Z = 3 X_1 + 2 X_2$

Subject To :

$$\begin{aligned} 2 X_1 + X_2 &\leq 3 \\ 3 X_1 + 4 X_2 &\leq 12 \\ X_1, X_2 &\geq 0 \end{aligned}$$

11 – In problem 10 (a) , let y_1 and y_2 be the dual variables. Determine whether the following pairs of primal-dual solutions are optimal:

(a) ($x_1 = 3$, $x_2 = 1$; $y_1 = 4$, $y_2 = 1$)

(b) ($x_1 = 4$, $x_2 = 1$; $y_1 = 1$, $y_2 = 0$)

(c) ($x_1 = 3$, $x_2 = 0$; $y_1 = 5$, $y_2 = 0$)

12 – Consider the following LP expressed in the standard form :

Maximize $Z = 5 X_1 + 12 X_2 + 4 X_3$

Subject To:

$$\begin{aligned} X_1 + 2 X_2 + X_3 + X_4 &= 10 \\ 2 X_1 - X_2 + 3 X_3 &= 2 \\ X_1, X_2, X_3, X_4 &\geq 0 \end{aligned}$$

Each of the following inverses is associated with a basic feasible solution and only one of them represents the optimal solution. Show how you can identify the optimal solution by using the optimal – dual relationships:

(a) (X_4, X_3) , $\begin{pmatrix} 1 & -1/3 \\ 0 & 1/3 \end{pmatrix}$

(b) (X_2, X_1) , $\begin{pmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{pmatrix}$

(c) (X_2, X_3) , $\begin{pmatrix} 3/7 & -1/7 \\ 1/7 & 2/7 \end{pmatrix}$